



$\angle ACO = 90^\circ$ (angle between radius and tangent)

In right $\triangle ACO$,

$AO^2 = AC^2 + OC^2$ (Pythagoras theorem)

$(2r)^2 = AC^2 + r^2$ (AO = diameter given)

$$4r^2 - r^2 = AC^2$$

$$AC^2 = 3r^2$$

$$AC = \sqrt{3} r$$

$\triangle ODC \sim \triangle OCA$ by AA corollary

$$\frac{DC}{CA} = \frac{OC}{OA}$$

$$\frac{DC}{\sqrt{3}r} = \frac{r}{2r}$$

$$DC = \frac{\sqrt{3}}{2} r$$

$$2DC = \sqrt{3} r$$

$$BC = \sqrt{3} r$$

Now in $\triangle ABC$ $AC = AB = BC = \sqrt{3} r$

Therefore $\triangle ABC$ is equilateral

