Worksheet : Class XI

Trigonommetric Functions - Synopsis

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Relation between Radian and Degree Measure

$$Radian\ measure = \frac{\pi}{180} \times Degree\ measure; \qquad Degree\ measure = \frac{180}{\pi} \times Radian\ \ measure.$$

$$1^0 = 60'; \qquad 1' = 60''.$$

Domain and range of Trigonometric Functions.

Trigonometric Functions	Domain	Range
$\sin x$	**R**	[-1, 1]
$\cos x$	**R**	[-1, 1]
$\tan x$	$R - \left\{x : x = \frac{(2n+1)\pi}{2},\ n\ \varepsilon Z\right\}$	**R**
$\cot x$	$R - \{x : x = n\pi,\ n\ \varepsilon Z\}$	**R**
$\sec x$	$R - \left\{x : x = \frac{(2n+1)\pi}{2}\right\},\ n\ \varepsilon Z$	**R** - [-1, 1]
$\operatorname{cosec} x$	$R - \{x : x = n\pi,\ n\ \varepsilon Z\}$	**R** - [-1, 1]

Sign of Trigonometric Functions in Different Quadrants.

Trigonometric Functions	QI	QII	QIII	QIV
$\sin x$	+ve	+ve	-ve	-ve
$\cos x$	+ve	-ve	-ve	+ve
$\tan x$	+ve	-ve	+ve	-ve
$\cot x$	+ve	-ve	+ve	-ve
$\sec x$	+ve	-ve	-ve	+ve
$\operatorname{cosec} x$	+ve	+ve	-ve	-ve

Note:- To remember :- **ASTC**(**A**ll **S**tudents **T**ake **C**offee)- representing the positive quadrants.

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Table Values of Trigonometric Functions.

Functions	0^0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0

Trigonometric Identities.

$$\cos ecx = \frac{1}{\sin x}. \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}. \qquad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}.$$

$$\cos^2 x + \sin^2 x = 1; \qquad 1 + \tan^2 x = \sec^2 x. \qquad 1 + \cot^2 x = \cos ec^2 x.$$

Trigonometric Ratio of Allied Angles.

- $\sin(-x) = -\sin x.$
- $\cos(-x) = \cos x.$
- $\tan(-x) = -\tan x.$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x.$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x.$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x.$
- $\sin\left(\frac{\pi}{2} + x\right) = \cos x.$

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- $\cos(\pi - x) = -\cos x.$
- $\sin(\pi - x) = \sin x.$
- $\cos(\pi + x) = -\cos x.$
- $\sin(\pi + x) = -\sin x.$
- $\cos(2\pi - x) = \cos x.$
- $\sin(2\pi - x) = -\sin x.$
- $\cos(2n\pi + x) = \cos x, \quad n \varepsilon Z.$
- $\sin(2n\pi + x) = \sin x, \quad n \varepsilon Z.$

Sum and Difference of Two Angles.

- $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y.$
- $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y.$
- If none of the angles $x,\ y$ and $(x \pm y)$ is an odd multiple of $\frac{\pi}{2}$, then

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$ and

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

- If none of the angles $x,\ y$ and $(x \pm y)$ is a multiple of π, then

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x},$$ and

$$\cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}.$$

Trigonometric Ratio of Multiple Angles.

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$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$= \frac{1-\tan^2 x}{1+\tan^2 x}$$

Note:- $\cos^2 x = \frac{1+\cos 2x}{2}$ and $\sin^2 x = \frac{1-\cos 2x}{2}$.

- $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}.$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}.$
- $\sin 3x = 3\sin x - 4\sin^3 x.$
- $\cos 3x = 4\cos^3 x - 3\cos x.$
- $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

Some More Trigonometric Identities.

- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}.$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}.$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}.$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}.$
- $2\cos x\cos y = \cos(x+y) + \cos(x-y).$
- $-2\sin x\sin y = \cos(x+y) - \cos(x-y).$
- $2\sin x\cos y = \sin(x+y) + \sin(x-y).$
- $2\cos x\sin y = \sin(x+y) - \sin(x-y).$

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General Solutions of Trigonometric Equations.

- $\sin x = 0 \Rightarrow x = n\pi,$ where $n \varepsilon Z$
- $\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2},$ where $n \varepsilon Z$
- $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y,$ where $n \varepsilon Z$
- $\cos x = \cos y \Rightarrow x = 2n\pi \pm y,$ where $n \varepsilon Z$
- $\tan x = \tan y \Rightarrow x = n\pi + y,$ where $n \varepsilon Z$

Note:- The solution of a trigonometric equation for which $0 \le x < 2\pi$ are called ***principal Solution.***

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