



To prove  $ar(DOC) = ar(AOB)$   
 $ar(DCB) = ar(ACB)$   
 $DA \parallel CB$ ,  $ABCD$  is a parallelogram  
 Construct - draw  $DP \perp AC$ ,  $BQ \perp AC$

Proof -  $\triangle DPO \cong \triangle BQO$  by AAS cor.  $\left[ \begin{array}{l} \angle 1 = \angle 2 = 90^\circ \\ \angle 3 = \angle 4 \text{ vert opp} \\ OD = OB \text{ given} \end{array} \right.$

$\therefore DP = BQ$  (cpct)

$$ar(\triangle DPO) = ar(\triangle BQO) \dots \textcircled{1}$$

$\triangle CPD \cong \triangle AQB$  by RHS property  $\left[ \begin{array}{l} \angle 6 = \angle 5 = 90^\circ \\ CD = AB \text{ given} \\ DP = BQ \text{ proved} \end{array} \right.$

$\angle 7 = \angle 8$  (cpct)

But these are alternate interior angles

$\therefore CD \parallel BA$

$$ar(\triangle CPD) = ar(\triangle AQB) \dots \textcircled{2}$$

$\textcircled{1} + \textcircled{2}$

$$ar(\triangle DPO) + ar(\triangle CPD) = ar(\triangle BQO) + ar(\triangle AQB)$$

$$\Rightarrow ar(DOC) = ar(AOB)$$

$$ar(\triangle DOC) + ar(\triangle BOC) = ar(\triangle AOB) + ar(\triangle BOC)$$

$$\Rightarrow ar(\triangle DCB) = ar(\triangle ABC)$$

$\Rightarrow DA \parallel CB$  [ $\Delta$ s equal in area and on same base]

$\square ABCD$  is a parallelogram [ $DA \parallel CB$   
 $CD \parallel BA$ ]