

To Prove  $AC = BD$   
 $OA = OC, OB = OD$   
 $AC \perp BD$

Proof In  $\triangle DAB$  and  $\triangle CBA$

$DA = CB$  (Sides of Square)

$\angle 1 = \angle 2 = 90^\circ$

$AB = BA$  (Common)

$\therefore \triangle DAB \cong \triangle CBA$  by SAS prop.

$BD = AC$  (Cpct)

In  $\triangle AOD$  and  $\triangle COB$

$\angle 3 = \angle 4$  [alternate in. angles,  $AD \parallel BC$ ]

$AD = CB$  (Sides of Square)

$\angle 5 = \angle 6$  [alternate in. angles,  $AD \parallel BC$ ]

$\therefore \triangle AOD \cong \triangle COB$  by ASA prop.

$OA = OC$  (Cpct)

$OD = OB$  (Cpct)

In  $\triangle AOD$  and  $\triangle AOB$

$OA = OA$  (Common)

$OD = OB$  (proved)

$AD = AB$  (Sides of Square)

$\therefore \triangle AOD \cong \triangle AOB$  by SSS prop.

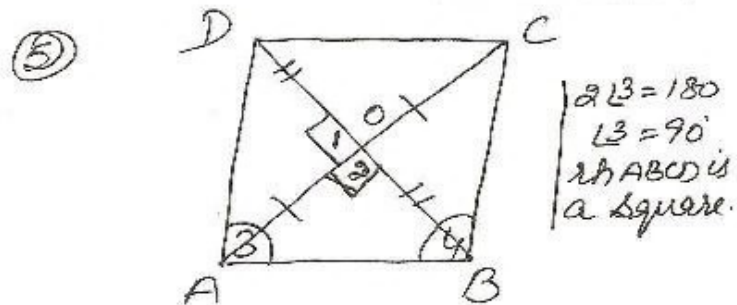
$\angle 7 = \angle 8$  (Cpct)

But  $\angle 7 + \angle 8 = 180^\circ$  (Linear pair axiom)

$2\angle 7 = 180$

$\Rightarrow \angle 7 = 90^\circ$

$\Rightarrow AC \perp BD$



$2\angle 1 = 180$   
 $\angle 1 = 90^\circ$   
 $\therefore \square ABCD$  is a square.

To Prove -  $\square ABCD$  is a square.

Proof - In  $\triangle AOD$  and  $\triangle AOB$

$OA = OA$  (Common)

$\angle 1 = \angle 2 = 90^\circ$

$OD = OB$  (given)

$\therefore \triangle AOD \cong \triangle AOB$  by SAS prop.

$AD = AB$  (Cpct)

Similarly

$AB = BC$  ... (i)

$BC = CD$  ... (ii)

From (i), (ii), (iii)

$AB = BC = CD = DA$

$\Rightarrow \square ABCD$  is a rhombus

In  $\triangle DAB$  and  $\triangle CBA$

$DA = CB$  (Proved)

$AB = BA$  (Common)

$BD = AC$  (given)

$\therefore \triangle DAB \cong \triangle CBA$  by SSS prop.

$\angle 3 = \angle 4$  (Cpct)

But  $\angle 3 + \angle 4 = 180^\circ$  (Co-linear)