1) i

let $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 2 - 2$

$= 0$

$\cancel{\because x = 0}$

$\because$ remainder $= 0$

$\therefore$ $x+1$ is a factor of $p(x)$ by factor theorem

1(ii) let $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1$

$= 1$

$\because$ remainder $\neq 0$

$\therefore$ $x+1$ is not a factor of $p(x)$

1(iii) let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 - 1 + 1$

$= 1 - \cancel{3} + \cancel{3} - \cancel{1} + \cancel{1}$

$= 1$

$\because$ remainder $\neq 0$

$\therefore$ $x+1$ is a factor of $p(x)$

1(iv) let

$p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2}$

$\because$ remainder $\neq 0$, not a factor

2(i) $p(x) = 2x^3 + x^2 - 2x - 1$

$g(x) = x + 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1$

$= 3 - 3$

$= 0$

$\because$ remainder $= 0$

$\therefore$ $g(x)$ is a factor of $p(x)$

2(ii) $p(x) = x^3 + 3x^2 + 3x + 1$

$g(x) = x + 2$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= 13 - 14$

$= -1$

$\because$ remainder $\neq 0$

$\therefore$ $g(x)$ is not a factor of $p(x)$

2(iii) $p(x) = x^3 - 4x^2 + x + 6$

$g(x) = x - 3$

$p(3) = 3^3 - 4 \times 3^2 + 3 + 6$

$= 27 - 36 + 9$

$= 36 - 36$

$= 0$

$\because$ remainder $= 0$

$\therefore$ $g(x)$ is a factor of $p(x)$ by factor theorem.