

①

Suppose  $\sqrt{5}$  is rational

let  $\sqrt{5} = \frac{p}{q}$  where  $p, q$  are integers  
and  $q \neq 0$

dividing  $p, q$  by their H.C.F

$\sqrt{5} = \frac{a}{b}$ ,  $a, b$  are coprime

$$\Rightarrow a = b\sqrt{5}$$

Squaring both sides

$$a^2 = 5b^2 \dots \textcircled{i}$$

$\Rightarrow 5$  is a factor of  $a^2$

$\Rightarrow 5$  is a factor of  $a$

$\Rightarrow a = 5c$  where  $c$  is any integer

Squaring both sides

$$a^2 = 25c^2 \dots \textcircled{ii}$$

From  $\textcircled{i}$  and  $\textcircled{ii}$

$$5b^2 = 25c^2$$

$$\Rightarrow b^2 = 5c^2$$

$\Rightarrow 5$  is a factor of  $b^2$

$\Rightarrow 5$  is a factor of  $b$

$\therefore 5$  is a common factor of  $a$  and  $b$

but this is contradiction

$\because a, b$  are coprime

$\therefore$  our supposition is false

$\sqrt{5}$  is irrational