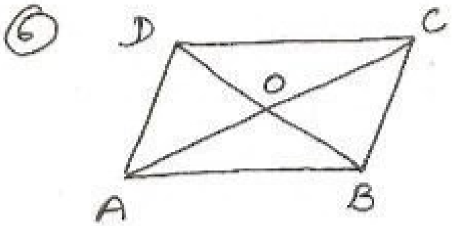
(6)



given - In fig. ▱ABCD is a ||gm

To Prove - $AB^2+BC^2+CD^2+DA^2 = AC^2+BD^2$

Proof - diagonals of ||gm bisect each other

$\therefore OA = OC \cdots$ (i)

$OB = OD \cdots$ (ii)

In $\triangle DAC$, DO is median to side BC

$\therefore CD^2+DA^2 = 2DO^2+\frac{AC^2}{2} \cdots$ (iii)

BO is median to side BC of $\triangle BAC$

$\therefore AB^2+BC^2 = 2BO^2+\frac{AC^2}{2} \cdots$ (iv)

(iii) + (iv)

$$AB^2+BC^2+CD^2+DA^2 = \frac{AC^2}{2}+\frac{AC^2}{2}+2DO^2+2BO^2$$

$$= \frac{\cancel{2}AC^2}{\cancel{2}} + \cancel{2}\times\frac{BD^2}{\cancel{4}\,2} + \cancel{2}\times\frac{BD^2}{\cancel{4}\,2}$$

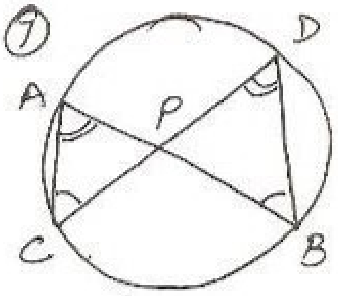
$[\because BO = DO = \frac{1}{2}BD]$

$$= AC^2 + \frac{BD^2}{2} + \frac{BD^2}{2}$$

$$= AC^2 + \cancel{2}\frac{BD^2}{\cancel{2}}$$

$\therefore AB^2+BC^2+CD^2+DA^2 = AC^2+BD^2$

(7)



To Prove (i) $\triangle APC \sim \triangle DPB$

(ii) $AP \cdot PB = CP \cdot DP$

Proof - $\angle C = \angle B$ [angles in Same Seg.]

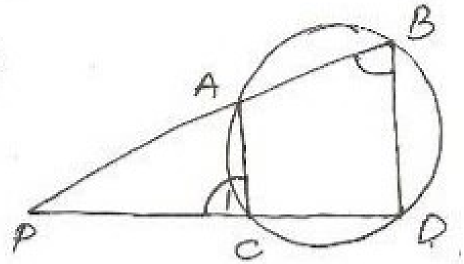
$\angle A = \angle D$

$\therefore \triangle APC \sim \triangle DPB$ by AA cor.

$$\Rightarrow \frac{AP}{DP} = \frac{CP}{BP}$$

$$\Rightarrow AP \cdot PB = CP \cdot DP$$

(8)



To Prove $\triangle APC \sim \triangle DPB$

$PA \cdot PB = PC \cdot PD$

Proof $\angle P = \angle P$

$\angle 1 = \angle B$ [exterior angle prop. of cyclic quad.]

$\therefore \triangle PAC \sim \triangle PDB$ by AA cor.

$$\frac{PA}{PD} = \frac{PC}{PB}$$

$$\Rightarrow PA \cdot PB = PC \cdot PD$$