

given - In fig $\triangle ABC \sim \triangle DEF$
 $ar(\triangle ABC) = ar(\triangle DEF)$

To Prove $\triangle ABC \cong \triangle DEF$

Proof $\triangle ABC \sim \triangle DEF$

$$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2}$$

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[$\because ar(\triangle ABC) = ar(\triangle DEF)$]

$$\text{Sim. } \frac{EF}{BA} = \frac{1}{2} \dots \text{ (ii)}$$

$$\Rightarrow AB^2 = DE^2$$

$$\Rightarrow AB = DE$$

$$\begin{aligned} \angle A &= \angle D \\ \angle B &= \angle E \end{aligned} \quad (\triangle ABC \sim \triangle DEF)$$

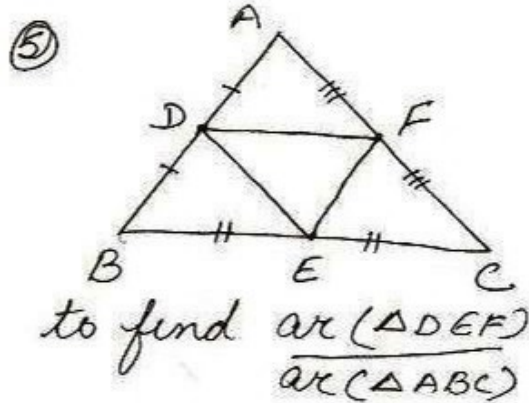
In $\triangle ABC$ and $\triangle DEF$

$$\angle A = \angle D$$

$$AB = DE \quad (\text{Proved})$$

$$\angle B = \angle E$$

$\therefore \triangle ABC \cong \triangle DEF$ by ASA prop.



to find $\frac{ar(\triangle DEF)}{ar(\triangle ABC)}$

Solution

In $\triangle ABC$, DF joins midpts of sides AB and AC respectively

$$DF = \frac{1}{2} BC \quad [\text{Midpt. Theorem}]$$

$$\Rightarrow \frac{DF}{BC} = \frac{1}{2} \dots \text{ (i)}$$

$$\frac{DE}{AC} = \frac{1}{2} \dots \text{ (iii)}$$

From (i), (ii), (iii)

$$\frac{DF}{BC} = \frac{EF}{BA} = \frac{DE}{AC}$$

$\therefore \triangle DEF \sim \triangle ABC$ by SSS prop

$$\begin{aligned} \therefore \frac{ar(\triangle DEF)}{ar(\triangle ABC)} &= \left(\frac{DF}{BC}\right)^2 \\ &= \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} \end{aligned}$$

$$\therefore \frac{ar(\triangle DEF)}{ar(\triangle ABC)} = \frac{1}{4}$$