

$$\begin{aligned}
 5 \text{ (x)} \quad & \frac{1 + \tan^2 A}{1 + \cot^2 A} \\
 &= \frac{\sec^2 A}{\operatorname{cosec}^2 A} \\
 &= \frac{1}{\cos^2 A} \cdot \frac{1}{\frac{1}{\sin^2 A}} \\
 &= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} \\
 &= \tan^2 A
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 \\
 &= \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2 \\
 &= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2 \\
 &= \frac{\cancel{(1 - \tan A)}^2 \tan^2 A}{(\cancel{\tan A - 1})^2} \\
 &= \tan^2 A
 \end{aligned}$$

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$