

① i

$$\text{let } p(x) = x^3 + x^2 + x + 1$$

$$\begin{aligned} p(-1) &= (-1)^3 + (-1)^2 + (-1) + 1 \\ &= -1 + 1 - 1 + 1 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

\therefore remainder = 0

\therefore $x+1$ is a factor of $p(x)$ by factor theorem

1① let $p(x) = x^4 + x^3 + x^2 + x + 1$

$$\begin{aligned} p(-1) &= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1 \\ &= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1 \\ &= 1 \end{aligned}$$

\therefore remainder $\neq 0$

\therefore $x+1$ is not a factor of $p(x)$

1② let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$$\begin{aligned} p(-1) &= (-1)^4 + 3(-1)^3 + 3(-1)^2 - 1 + 1 \\ &= 1 - \cancel{3} + \cancel{3} - \cancel{1} + \cancel{1} \\ &= 1 \end{aligned}$$

\therefore remainder $\neq 0$

\therefore $x+1$ is a factor of $p(x)$

1③ let

$$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$$

$$\begin{aligned} p(-1) &= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} \\ &= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

\therefore remainder $\neq 0$, not a factor

2① $p(x) = 2x^3 + x^2 - 2x - 1$

$$g(x) = x + 1$$

$$\begin{aligned} p(-1) &= 2(-1)^3 + (-1)^2 - 2(-1) - 1 \\ &= -2 + 1 + 2 - 1 \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

\therefore remainder = 0

\therefore $g(x)$ is a factor of $p(x)$

2② $p(x) = x^3 + 3x^2 + 3x + 1$

$$g(x) = x + 2$$

$$\begin{aligned} p(-2) &= (-2)^3 + 3(-2)^2 + 3(-2) + 1 \\ &= -8 + 12 - 6 + 1 \\ &= 13 - 14 \\ &= -1 \end{aligned}$$

\therefore remainder $\neq 0$

\therefore $g(x)$ is not a factor of $p(x)$

2③ $p(x) = x^3 - 4x^2 + x + 6$

$$g(x) = x - 3$$

$$\begin{aligned} p(3) &= 3^3 - 4 \times 3^2 + 3 + 6 \\ &= 27 - 36 + 9 \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

\therefore remainder = 0

\therefore $g(x)$ is a factor of $p(x)$ by factor theorem.