

Ex 5.4

$$\begin{aligned} \textcircled{2} \quad a_3 + a_7 &= 6 \\ a + 2d + a + 6d &= 6 \\ \Rightarrow 2a + 8d &= 6 \\ \Rightarrow a + 4d &= 3 \\ \Rightarrow a &= 3 - 4d \dots \textcircled{1} \end{aligned}$$

$$\begin{aligned} a_3 \times a_7 &= 8 \\ (a + 2d)(a + 6d) &= 8 \\ \text{using } \textcircled{1} \\ (3 - 4d + 2d)(3 - 4d + 6d) &= 8 \\ \Rightarrow (3 - 2d)(3 + 2d) &= 8 \\ \Rightarrow 3^2 - (2d)^2 &= 8 \\ \Rightarrow 9 - 4d^2 &= 8 \\ \Rightarrow 4d^2 &= 1 \\ \Rightarrow d &= \pm \sqrt{\frac{1}{4}} \\ &= \pm \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{if } d &= \frac{1}{2} \\ a &= 3 - 4 \times \frac{1}{2} \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} S_{16} &= \frac{16}{2} \left[2 \times 1 + 15 \times \frac{1}{2} \right] \\ &= 8 (2 + 7.5) \\ &= 8 \times 9.5 \end{aligned}$$

$$\begin{aligned} \text{if } d &= -\frac{1}{2} \\ a &= 3 - 4 \times -\frac{1}{2} \\ &= 3 + 2 \\ &= 5 \\ S_{16} &= \frac{16}{2} \left[2 \times 5 + 15 \times -\frac{1}{2} \right] \\ &= 8 (10 - 7.5) \\ &= 8 \times 2.5 \\ &= 20 \\ S_{16} &= 20, 76 \end{aligned}$$