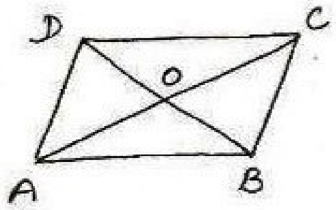
⑥



given - In fig. ▱ABCD is a ||gm

To Prove - $AB^2+BC^2+CD^2+DA^2 = AC^2+BD^2$

Proof - diagonals of ||gm bisect each other

∴ $OA = OC$ ... (i)
$OB = OD$ ... (ii)

In $\triangle DAC$, DO is median to side BC

∴ $CD^2+DA^2 = 2DO^2+\frac{AC^2}{2}$ ...(iii)

BO is median to side BC of $\triangle BAC$

∴ $AB^2+BC^2 = 2BO^2+\frac{AC^2}{2}$ ...(iv)

(iii) + (iv)

$AB^2+BC^2+CD^2+DA^2 = \frac{AC^2}{2}+\frac{AC^2}{2}+2DO^2+2BO^2$

$= \frac{\cancel{2}AC^2}{\cancel{2}} + \cancel{2}\times\frac{BD^2}{\cancel{4}2} + \cancel{2}\times\frac{BD^2}{\cancel{4}2}$

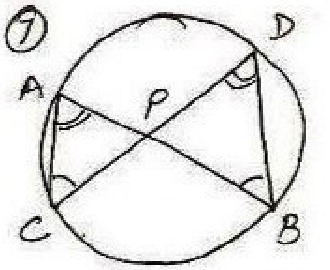
[∵ $BO = DO = \frac{1}{2}BD$]

$= AC^2 + \frac{BD^2}{2} + \frac{BD^2}{2}$

$= AC^2 + \cancel{2}\frac{BD^2}{\cancel{2}}$

∴ $AB^2+BC^2+CD^2+DA^2 = AC^2+BD^2$

⑦



To prove (i) $\triangle APC \sim \triangle DPB$
(ii) $AP.PB = CP.DP$

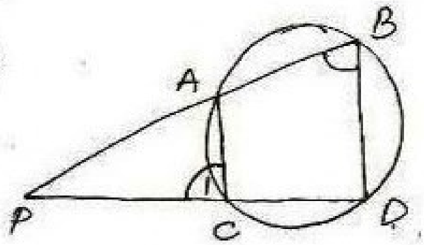
Proof - $\angle C = \angle B$ [angles in Same Seg.]
$\angle A = \angle D$

∴ $\triangle APC \sim \triangle DPB$ by AA cor.

⇒ $\frac{AP}{DP} = \frac{CP}{BP}$

⇒ $AP.PB = CP.DP$

⑧



To Prove $\triangle APC \sim \triangle DPB$
$PA.PB = PC.PD$

Proof $\angle P = \angle P$
$\angle 1 = \angle B$ [exterior angle prop. of cyclic quad.]

∴ $\triangle PAC \sim \triangle PDB$ by AA cor.

$\frac{PA}{PD} = \frac{PC}{PB}$

⇒ $PA.PB = PC.PD$