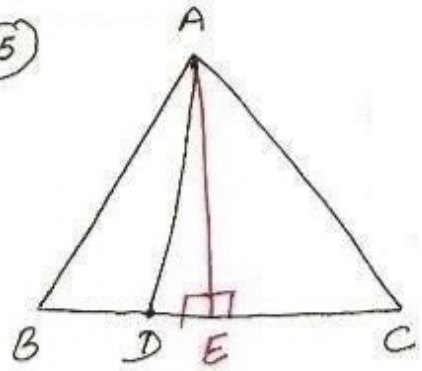
(15)



given - In $\triangle ABC$
$AB = BC = CA$,
$BD = \frac{1}{3} BC$

To Prove - $9AD^2 = 7AB^2$

Const. $AE \perp BC$

Proof - In equilateral $\triangle ABC$, $AE \perp BC$

$BE = CE$ ...(i)

[In an equilateral triangle altitude is also median]

In rt $\triangle AEB$

$AB^2 = AE^2 + BE^2$ ...(ii) [Pythagoras Theorem]

In rt $\triangle AED$

$AD^2 = AE^2 + DE^2$ ...(iii) [Pythagoras Theorem]

(ii) - (iii)

$AB^2 - AD^2 = BE^2 - DE^2$

$= (BE - DE)(BE + DE)$

$= BD(CE + DE)$ ($\because BE = DE$)

$= BD \times CD$

$= \frac{1}{3} BC \times \frac{2}{3} BC$ [$CD = BC - BD = BC - \frac{1}{3} BC = \frac{3BC - BC}{3} = \frac{2}{3} BC$]

$= \frac{2}{9} BC^2$

$AB^2 - AD^2 = \frac{2}{9} AB^2$ [$\because AB = BC$]

($\times 9$)

$9AB^2 - 9AD^2 = 2AB^2$

$\Rightarrow 7AB^2 = 9AD^2$