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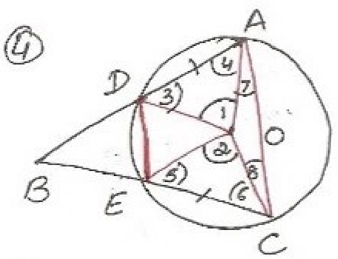

To prove $\angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$

Proof $AD = CE$

$\Rightarrow \angle 1 = \angle 2$ [equal chords sub. equal angles at the centre of ⊙]

In $\triangle OAD$ and $\triangle OCE$

$OA = OC$ [radii of same ⊙]

$\angle 1 = \angle 2$ [Proved]

$OD = OE$ [radii of same ⊙]

$\therefore \triangle OAD \cong \triangle OCE$ by SAS prop

$\angle 4 = \angle 6$ (cpct)

In $\triangle OAC$, $OA = OC$

$\angle 8 = \angle 7$

In $\triangle ABC$

$\angle B + \angle BAC + \angle BCA = 180^\circ$ [angle sum prop of $\triangle$]

$\angle B + \angle 4 + \angle 7 + \angle 6 + \angle 8 = 180^\circ$

$\angle B + \angle 4 + \angle 4 + \angle 7 + \angle 8 = 180^\circ$

$\angle B + \angle 3 + \angle 4 + (180^\circ - \angle AOC) = 180^\circ$

$\angle B + 180^\circ - \angle 1 + \cancel{180} - \angle AOC = \cancel{180}$

$\angle B = \angle 1 + \angle AOC - 180^\circ$

$(\times 2)$

$2\angle B = 2\angle 1 + 2\angle AOC - 360$

$= \angle 1 + \angle 2 + \angle AOC + \angle AOC - 360^\circ$ [since $\angle 1 = \angle 2$]

$= \cancel{360^\circ} - \cancel{\angle AOC} - \angle DOE + \cancel{\angle AOC}$ [$\angle 1 + \angle 2 + \angle AOC + \angle DOE = 360^\circ$]

$+ \angle AOC - \cancel{360}$

$\Rightarrow \angle B = \frac{1}{2}(\angle AOC - \angle DOE)$

or

$\angle AOC = \angle 9$

$\angle DOE = \angle 10$

In $\triangle ABC$

$\angle B + \angle BAC + \angle BCA = 180^\circ$

$\angle B + \angle 4 + \angle 7 + \angle 6 + \angle 8 = 180^\circ$

But $\angle 7 + \angle 8 = 180^\circ - \angle 9$ [angle sum prop of $\triangle$]

$\angle B + \angle 4 + \angle 6 + \cancel{180^\circ} - \angle 9 = \cancel{180^\circ}$

$\Rightarrow 2\angle B + 2\angle 4 + 2\angle 6 = 2\angle 9$

$\Rightarrow 2\angle B + \angle 4 + \angle 3 + \angle 5 + \angle 6 = 2\angle 9$

[$\because \angle 3 = \angle 4$, $\angle 5 = \angle 6$ (isos. $\triangle$ prop)]

But $\angle 3 + \angle 4 = 180^\circ - \angle 1$

and $\angle 5 + \angle 6 = 180^\circ - \angle 2$

$2\angle B + 180 - \angle 1 + 180^\circ - \angle 2 = 2\angle 9$

$2\angle B + 360 = 2\angle 9 + \angle 1 + \angle 2$

$2\angle B + 360^\circ = \angle 9 + \angle 9 + \angle 1 + \angle 2$

But $\angle 1 + \angle 2 + \angle 9 = 360^\circ - \angle 10$

$2\angle B + \cancel{360} = \angle 9 + \cancel{360} - \angle 10$

$\Rightarrow \angle B = \frac{1}{2}(\angle 9 - \angle 10)$

$\Rightarrow \angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$