Worksheet : Class XII
Matrices & Determinants

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MATHEMATICS WORKSHEET – I

Name:

ALGEBRA (Matrices and Determinants)– 13 Marks

March 2012

1. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. [1]

2. If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, write the value of x. [1]

3. Simplify: $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$. [1]

4. Using properties of determinants, prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$$ [4]

5. Using matrices, solve the following system of linear equations. [6]

$x - y + 2z = 7; \quad 3x + 4y - 5z = -5; \quad 2x - y + 3z = 12;$

OR

Using elementary operations, find the inverse of the following matrix:

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}.$$

Worksheet : Class XII
Matrices & Determinants

March 2011

6. For a 2 x 2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}. [1]

7. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? [1]

8. Write A^{-1} for $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$. [1]

9. Using properties of determinants, prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$ [4]

OR

Using properties of determinants, solve the following for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 2x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$$

10. Using matrix method, solve the following system of equations:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4; \quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1; \quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2; \quad x, y, z \neq 0.$$ [6]

OR

Using elementary transformation, find the inverse of the matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

Worksheet : Class XII
Matrices & Determinants

Foreign 2010 (SET 1)

11. Write a square matrix of order 2, which both symmetric and skew symmetric. [1]

12. What is the value of the following determinant? [1]

$$\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$$

13. From the following matrix equation, find the value of the x: [1]

$$\begin{pmatrix} x+y & 4 \\ -5 & 3y \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -5 & 6 \end{pmatrix}$$

14. Prove the following, using properties of determinants:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3. \quad [4]$$

OR

Find the inverse of $A = \begin{pmatrix} 3 & -1 \\ -4 & 1 \end{pmatrix}$ using elementary transformations.

15. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}. Using A^{-1}. Solve the following system of equations: [6]

$$2x - 3y + 5z = 16;\ \ 3x + 2y - 4z = -4;\ \ x + y - 2z = -3$$

Foreign 2010 (SET 2)

16. From the following matrix equation, find the value of x: [1]

$$\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

17. Prove the following, using properties of determinants: [4]

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$$

Worksheet : Class XII
Matrices & Determinants

OR

Find the inverse of the following matrix, using elementary transformations: $A=\begin{pmatrix}3 & 2\\ 7 & 5\end{pmatrix}$

18. Using matrices, solve the following system of equations: [6]

$$3x-2y+3z=-1;\quad 2x+y-z=6;\quad 4x-3y+2z=5.$$

Foreign 2010 (SET 3)

19. If $\begin{pmatrix}3 & 4\\ 2 & x\end{pmatrix}\begin{pmatrix}x\\ 1\end{pmatrix}=\begin{pmatrix}19\\ 15\end{pmatrix}$, find the value of x. [1]

20. Prove the following, using properties of determinants: [4]

$$\begin{vmatrix}a+bx^2 & c+dx^2 & p+qx^2\\ ax^2+b & cx^2+d & px^2+q\\ u & v & w\end{vmatrix}=(x^4-1)\begin{vmatrix}b & d & q\\ a & c & p\\ u & v & w\end{vmatrix}$$

All India 2010 (SET 1)

21. What positive value of x makes the following pair of determinants equal? [1]

$$\begin{vmatrix}2x & 3\\ 5 & x\end{vmatrix},\ \begin{vmatrix}16 & 3\\ 5 & 2\end{vmatrix}$$

22. Write the adjoint of the following matrix: [1]

$$\begin{pmatrix}2 & -1\\ 4 & 3\end{pmatrix}$$

23. A square matrix of order 3 and $|A|=7$. Write the value of $|adj.A|$. [1]

24. Express the following as the sum of a symmetric and skew symmetric matrix, and verify your result:

$$\begin{pmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{pmatrix}$$ [4]

25. Using properties of determinants, prove the following: [6]

$$\begin{vmatrix}x & x^2 & 1+px^3\\ y & y^2 & 1+py^3\\ z & z^2 & 1+pz^3\end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x).$$

OR

Worksheet : Class XII
Matrices & Determinants

Find the inverse of the following matrix using elementary operations:

$$A=\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

All India 2010 (SET 2)

26. If $A=\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, then find the value of $A^2-3A+2I$. [4]

All India 2010 (SET 3)

27. For the following matrices A and B, verify that $(AB)^I = B^I A^I$. [4]

$$A=\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix},\ B=\begin{pmatrix} -1 & 2 & 1 \end{pmatrix}$$

28. Using matrices, solve the following system of equations: [6]

$x+2y-3z=-4;\quad 2x+3y+2z=2;\quad 3x-3y-4z=11.$

OR

If a,b,c are positive and unequal, show that the following determinant is negative

$$\Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Delhi 2010 (SET 1)

29. If $A=\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix? [1]

Worksheet : Class XII
Matrices & Determinants

30. What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$? [1]

31. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of k. [1]

32. Using elementary row operations, find the inverse of the matrix: $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ [4]

33. Using properties of determinants , show the following: [6]

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

Delhi 2010 (SET 2)

Find the minor of the element of second row and third column (a_{23}) in the following determinant:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

[1]

Delhi 2010 (SET 3)

34. If A is a square matrix of order 3 and $|3A| = k|A|$, then write the value of k. [1]

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35. If $A = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$, find the value of $3|A|$? [1]

36. If A is an invertible matrix of order 3 and $|A| = 5$, then find the value of $|adjA|$? [1]

37. For what value of y is the matrix $A = \begin{pmatrix} y^2+6 & 2y \\ y+3 & 2 \end{pmatrix}$ singular? [1]

38. Using properties of determinants, prove the following [4]

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

Worksheet : Class XII
Matrices & Determinants

39. If $A=\begin{pmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{pmatrix}$, find A^{-1}. Using A^{-1}. Solve the following system of equations: [6]

$$2x-3y+5z=11;\ \ 3x+2y-4z=-5;\ \ x+y-2z=-3$$

EXTRA QUESTIONS

40. If $A=\begin{pmatrix}2 & 3\\ 5 & -2\end{pmatrix}$, show that $A^{-1}=\frac{1}{19}A$. [1]

41. If $A=\begin{pmatrix}3 & -5\\ -4 & 2\end{pmatrix}$, show that $A^2-5A-14I=0$ [1]

42. If $A=\begin{pmatrix}-1\\ 2\\ 3\end{pmatrix}$ and $B=\begin{pmatrix}-2 & -1 & -4\end{pmatrix}$, verify that $(AB)^T=B^TA^T$. [1]

43. If $A=\begin{pmatrix}2 & -1\\ 3 & 2\end{pmatrix}$ and $B=\begin{pmatrix}0 & 4\\ -1 & 7\end{pmatrix}$, find $3A^2-2B+I$ [1]

44. If $f(x)=x^2-5x+7$ and $A=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$,find $f(A)$ [1]

45. Construct a 2 x 3 matrix whose elements are $a_{ij}=\frac{(i-2j)^2}{2}$. [1]

46. Prove that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)=(bc+ca+ab+abc)$ [4]

47. Prove that $\begin{vmatrix}1 & a & a^3\\ 1 & b & b^3\\ 1 & c & c^3\end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$. [4]

48. Using properties of determinants, prove that:

$$\begin{vmatrix}3a & -a+b & -a+c\\ a-b & 3b & c-b\\ a-c & b-c & 3c\end{vmatrix}=3(a+b+c)(ab+bc+ca)$$ [4]

Worksheet : Class XII
Matrices & Determinants

49. Using properties of determinants, prove that:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\lambda-\beta)(\alpha+\beta+\gamma)$$ [4]

50. Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$ [4]

51. Using properties of determinants, prove that: [4]

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0; \text{where } \alpha, \beta. \gamma \text{ are in AP.}$$

52. Using matrices, solve the following system of equations:

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10; \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10; \quad \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13.$$ [6]

53. Solve the following system of equation using matrix method:

$$x-y=3; \quad 2x+3y+4z=17; \quad y+2z=7.$$ [6]

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