3(i) let $x = 0.\overline{6}$ ... (i)

multiplying both sides by 10 (∵ 1 digit repeats)

$10x = 6.\overline{6}$ ... (ii)

(ii) − (i)

$10x - x = 6.\overline{6} - 0.\overline{6}$

$\Rightarrow 9x = 6$

$\Rightarrow x = \frac{\cancel{6}^{2}}{\cancel{9}_{3}}$

$\Rightarrow x = \frac{2}{3}$

$\therefore 0.\overline{6} = \frac{2}{3}$

3(ii) let $x = 0.4\overline{7}$

Multiplying both sides by 10 [∵ 1 digit (4) without −]

$10x = 4.\overline{7}$ ... (i)

again Mul. both sides by 10 [∵ 1 digit (7) with −]

$100x = 47.\overline{7}$ ... (ii)

(ii) − (i)

$100x - 10x = 47.\overline{7} - 4.\overline{7}$

$\Rightarrow 90x = 43$

$\Rightarrow x = \frac{43}{90}$

$\therefore 0.4\overline{7} = \frac{43}{90}$

3(iii) let $x = 0.\overline{001}$ ... (i)

mul. both sides by 1000

$1000x = 1.\overline{001}$ ... (ii)

(ii) − (i)

$1000x - x = 1.00\overline{1} - 0.00\overline{1}$

$\Rightarrow 999x = 1$

$\Rightarrow x = \frac{1}{999}$

$\therefore 0.\overline{001} = \frac{1}{999}$